CHAPTER

11

HEIGHTS AND DISTANCES

Syllabus

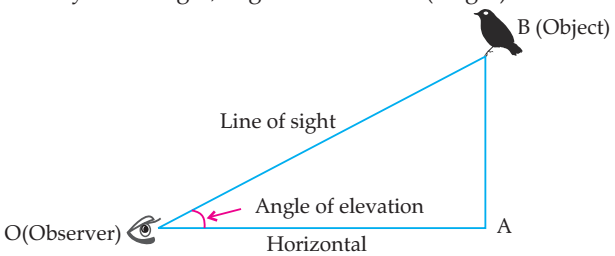
- **Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression 30°, 45°, 60° only.**

Trend Analysis

List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Heights and distances	1 Q (4 M)		1 Q (4 M)	2 Q (4 M)	1 Q (1 M) 2 Q (4 M)	1 Q (1 M) 2 Q (4 M)

Revision Notes

- The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.
- The angle of elevation of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, *i.e.*, the case when we raise our head to look at a point on the object. Now, we may identify line of sight, angles and altitude (height).

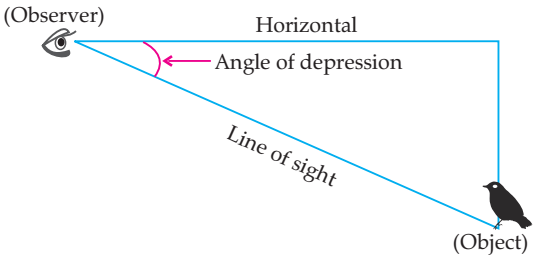


(i) ∠AOB is the angle of elevation.

(ii) The height AB, it means object is at point B from the ground at point A.

(iii) AO is the distance between the observer from the point A.

- The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, *i.e.*, the case when we lower our head to look at a point on the object.



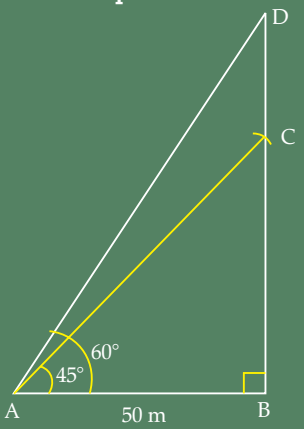
- The height of object above the water surface is equal to the depth of its image below the water surface.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

How is it done on the GREENBOARD?

Q.1. The angle of elevation of the top of a tower from a point 50 m away from the base of the tower is 45°. The angle of elevation of the top of the flag mounted on the tower is 60°. Find the height of the flag. U

Solution:

Step I: Let BC be the tower, CD be the flag and A is the point of observation.



Step II: First we will calculate the height of tower BC using triangle ABC, $\angle B = 90^\circ$

$$\tan 45^\circ = \frac{BC}{50}$$

$$\Rightarrow \quad 1 = \frac{BC}{50}$$

$$\Rightarrow \quad BC = 50 \text{ metre}$$

Step III: Calculation of height BD.

In ΔABD, $\angle B = 90^\circ$

$$\tan 60^\circ = \frac{BD}{50}$$

$$\Rightarrow \quad \sqrt{3} = \frac{BD}{50}$$

$$\Rightarrow \quad BD = 50\sqrt{3} \text{ metre}$$

Step IV: Calculation for height of flag

$$\text{Height of flag } CD = BD - BC$$

$$= 50\sqrt{3} - 50$$

$$\Rightarrow \quad CD = 50\left(\sqrt{3} - 1\right)$$

$$= 50\ (1.732 - 1)$$

$$= 50 \times 0.732$$

$$\Rightarrow \quad \text{Height of flag} = 36.6 \text{ metre}$$

Very Short Answer Type Questions — 1 mark each

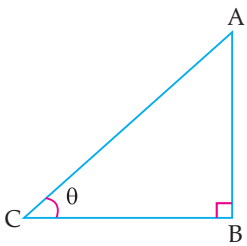
AI **Q. 1. The ratio of the length of a vertical rod and the length its shadow is 1 : $\sqrt{3}$. Find the angle of elevation of the sun at that moment ?**

A **[CBSE Delhi Set-I, 2020]**

Sol. Let AB be a vertical rod and BC be its shadow.

From the figure, $\angle ACB = \theta$.

In ΔABC,



$$\tan\theta = \frac{AB}{BC} \qquad \text{½}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \left[\because \frac{AB}{BC} = \frac{1}{\sqrt{3}} \text{(Given)}\right]$$

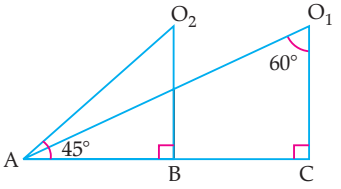
$$\Rightarrow \quad \tan\theta = \tan 30°$$

$$\Rightarrow \quad \theta = 30°$$

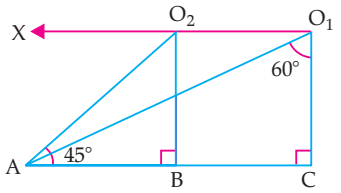
Hence, the angle of elevation of the sun is 30°. ½

AI **Q. 2. In the given figure, find the angles of depressions from the observing positions O_1 and O_2 respectively of the object A.**

A [CBSE OD Set-I, 2020]



Sol.



Draw $AC \parallel O_1X$ ½

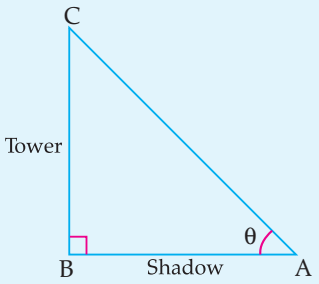
$\therefore \quad \angle AO_1X = 90° - 60° = 30°$

and $\angle AO_2X = \angle BAO_2 = 45°$. ½

Q. 3. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation of the Sun ?

U [CBSE Term-2, 2016]
[CBSE Delhi Set-I, II, 2017]

Sol.



Let the height of tower be AB and its shadow be BC.

$$\therefore \quad \frac{BC}{AB} = \tan\theta$$

$$= \frac{\sqrt{3}}{1}$$

$$= \tan 60°$$

Hence, the angle of elevation of Sun = 60°. **1**

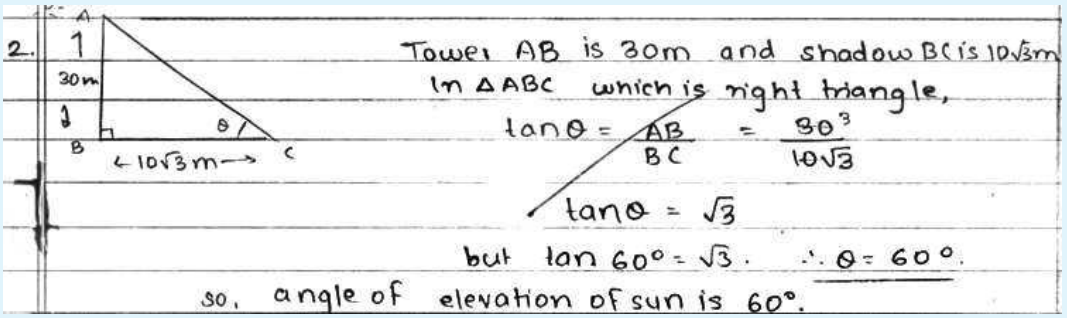
[CBSE Marking Scheme, 2017]

Q. 4. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ?

U [CBSE OD & Comptt. OD Set-I, II, III 2017] [Foreign Set-I, II, III, 2015]

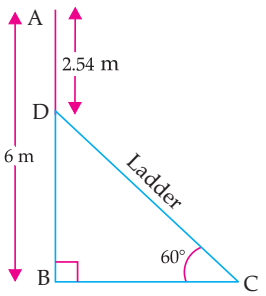
Topper Answer, 2017

Sol.



1

Q. 5. In the given figure, AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of pole. If AD = 2.54 m, find the length of the ladder. (use $\sqrt{3} = 1.73$)



C + A [CBSE Delhi Set I, II, III, 2016]

Sol. Given, $AD = 2.54$ m

$\therefore \quad DB = 6 - 2.54 = 3.46$ m

In ΔBCD, $\angle B = 90°$

$$\sin 60° = \frac{BD}{DC}$$

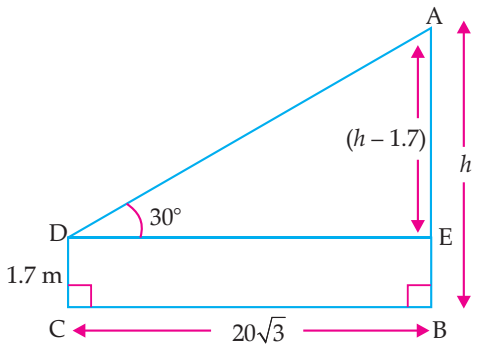
$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

$$\therefore \quad DC = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46 \times 2}{1.73} = 4 \text{ m}$$

$\therefore$ Length of ladder = 4 m. **1**

Q. 6. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of the tower. A [Foreign Set I, II, III, 2016]

Sol.



Let height of the tower AB be h metre

$\Rightarrow$ $AE = h - 1.7$

$BC = DE = 20\sqrt{3}$ m. (Given)

In ΔADE, $\angle E = 90°$

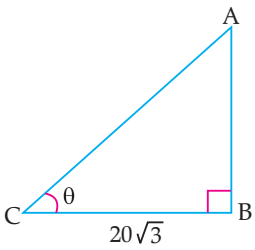
$$\tan 30° = \frac{h-1.7}{20\sqrt{3}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-1.7}{20\sqrt{3}}$$

$\Rightarrow$ $h - 1.7 = 20$

or $h = 20 + 1.7 = 21.7$ m **1**

AI **Q. 7. In figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the sun's altitude.**



C + A **[CBSE OD Set-I, II, III, 2015]**

Sol. Let the $\angle ACB$ be θ, $\angle B = 90°$

$$\tan \theta = \frac{AB}{BC}$$

$$\tan \theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30° \quad \mathbf{1}$$

$\Rightarrow$ $\theta = 30°$

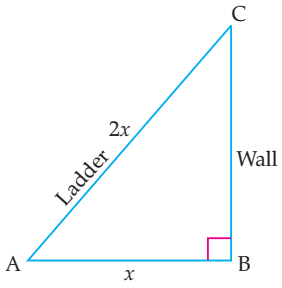
Thus, the sun's altitude is 30°.

[CBSE Marking Scheme, 2015]

Q. 8 If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal. A **[CBSE S.A.-2, 2015]**

Sol. Let the distance between the foot of the ladder and the wall, AB be x.

then AC, the length of the ladder $= 2x$



In ΔABC, $\angle B = 90°$

$$\cos A = \frac{x}{2x}$$

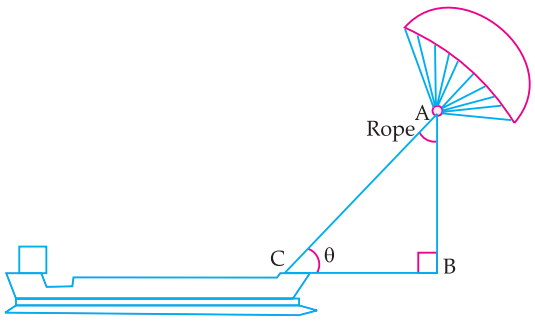
$$\Rightarrow \quad \cos A = \frac{1}{2} = \cos 60° \Rightarrow A = 60° \quad \mathbf{1}$$

Short Answer Type Questions-I — 2 marks each

Q. 1. 'Sky sails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Sky sails' technology allows the towing kite to gain a height of anything between 100 metres – 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the questions :

(i) In the given figure, if $\sin \theta = \cos (3\theta - 30°)$, where θ and $3\theta - 30°$ are acute angles, then find the value of θ.



(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200 m ? C **[CBSE SQP, 2020-21]**

Sol. (i) $\cos(90° - \theta) = \cos(3\theta - 30°)$

$\Rightarrow \quad 90° - \theta = 3\theta - 30°$

$\Rightarrow \quad \theta = 30°$ **1**

(ii) $\dfrac{AB}{AC} = \sin 30°$

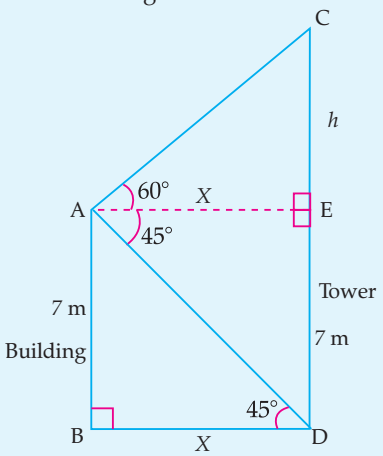
$\dfrac{200}{AC} = \dfrac{1}{2}$

$\therefore$ Length of rope $= AC = 400$ m **1**

[CBSE SQP Marking Scheme, 2020]

Q. 2. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. C + A **[CBSE Delhi Set-I, II, III, 2017]**

Sol. Let AB be building = 7 m



CD be the height of tower $= (7 + h)$ m ½

Let the distance BD be x m.

Then $BD = AE = x$ m

Now in ΔABD $\dfrac{AB}{BD} = \tan 45°$ ½

$\Rightarrow \quad \dfrac{7}{x} = 1$

$\Rightarrow \quad x = 7$ m ½

In ΔCEA $\dfrac{CE}{AE} = \tan 60°$

$\dfrac{h}{x} = \sqrt{3}$

$\Rightarrow \quad h = x\sqrt{3}$

Substituting the value of x we get

$h = 7\sqrt{3}$

$CD = CE + ED$

$= \left(7 + 7\sqrt{3}\right)$ m

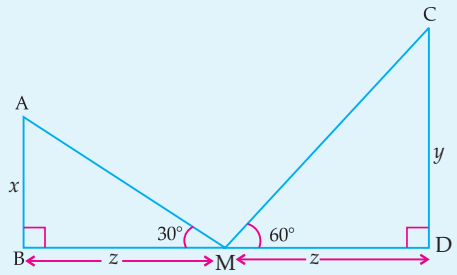
Hence, the height of tower $= 7\left(1 + \sqrt{3}\right)$ m. ½

[CBSE Marking Scheme, 2017]

Q. 3. The tops of two towers of height x and y, standing on the ground, subtend the angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

A **[CBSE Delhi Set-I, II, III, 2015]**

Sol.



Let M be the centre of the line joining their feet.

Let $BM = MD = z$

$\therefore \quad \tan\theta = \dfrac{\text{perpendicular}}{\text{base}}$

In ΔABM,

$\therefore \quad \dfrac{x}{z} = \tan 30°$

$\Rightarrow \quad x = z \times \dfrac{1}{\sqrt{3}}$ ...(i) ½

In ΔCDM,

$\dfrac{y}{z} = \tan 60°$

$y = z \times \sqrt{3}$ ...(ii) ½

From (i) and (ii), $\dfrac{x}{y} = \dfrac{z \times \frac{1}{\sqrt{3}}}{z \times \sqrt{3}}$

$\therefore \quad \dfrac{x}{y} = \dfrac{1}{3}$

$x : y = 1 : 3$ **1**

[CBSE Marking Scheme, 2015]

Q. 4. From the top of light house, 40 m above the water, the angle of depression of a small boat is 60°. Find how far the boat is from the base of the light house. U **[CBSE Board Term-2, 2015]**

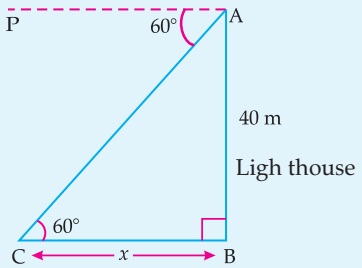
Sol. Let AB be the light house and C be the position of the boat.

Since, $\angle PAC = 60°$ $\therefore$ $\angle ACB = 60°$ **1**

Let BC be x m.

In ΔABC, $\dfrac{AB}{BC} = \tan 60°$

$\Rightarrow \quad \dfrac{40}{x} = \sqrt{3}$



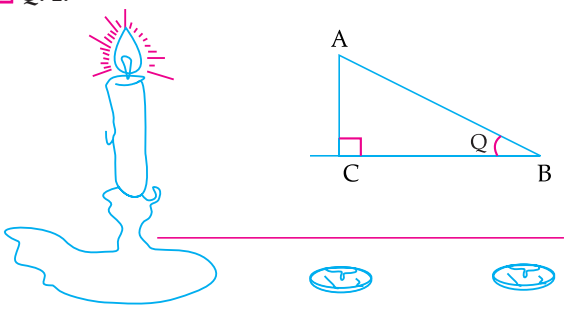
$\Rightarrow \quad x = \dfrac{40}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{40\sqrt{3}}{3}$ m

Hence, the boat is $\dfrac{40\sqrt{3}}{3}$ m away from the foot of the light house. **[CBSE Marking Scheme, 2015]** **1**

Short Answer Type Questions-II — 3 marks each

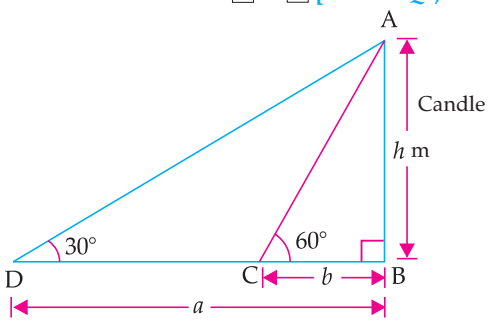
AI **Q. 1.**



If the angles of elevation of the top of the candle from two coins distant 'a' cm and 'b' cm ($a > b$) from its base and in the same straight line from it are 30° and 60°, then find the height of the candle.

C + A **[CBSE SQP, 2020-21]**

Sol.



½

Let AB = candle ½

C and D are two Points.

In ΔACB, $\tan 60° = \frac{AB}{BC} = \frac{h}{b}$

$\sqrt{3} = \frac{h}{b}$

$h = b\sqrt{3}$ ...(i) ½

In ΔADB, $\tan 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{a}$

$h = \frac{a}{\sqrt{3}}$ ...(ii) ½

Multiplying (i) and (ii), we get

$h^2 = b\sqrt{3} \times \frac{a}{\sqrt{3}}$ ½

$h^2 = ba$

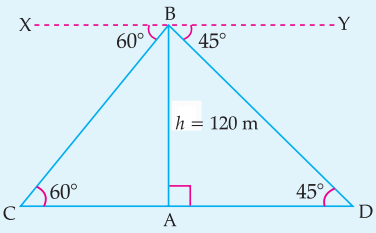
$h = \sqrt{ab}$ m ½

Hence, the height of the candle is $\sqrt{ab}$ m.

Q. 2. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of depression as 60° and 45°. Find the distance between two cars.

A **[Delhi/OD Compt. Set-I, II, III, 2017]**

Sol.



½

In ΔABD, $\angle ADB = \angle DBY = 45°$ (alternate angles) and in ΔABC, $\angle BCA = \angle XBC = 60°$

$\frac{AB}{AD} = \tan 45°$

$\frac{120}{AD} = 1$

$\Rightarrow$ $AD = 120$ m ...(i) **1**

Now, $\frac{AB}{CA} = \tan 60°$

$\frac{120}{CA} = \sqrt{3}$

$\Rightarrow$ $CA = \frac{120}{\sqrt{3}} = 40\sqrt{3}$ m ½

$CD = AD + CA$

$= 120 + 40\sqrt{3}$

$= 120 + 40 \times 1{\cdot}732$

$= 120 + 69{\cdot}28 = 189{\cdot}28$ m

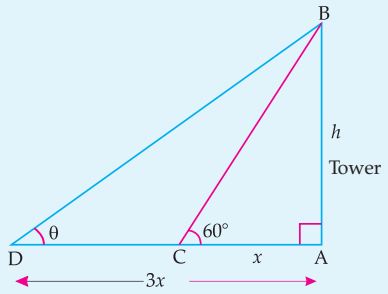
Hence the distance between two cars = 189·28 m. **1**

[CBSE Marking Scheme, 2017]

Q. 3. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the sun is 60°. Find the angle of elevation of the sun at the time of the longer shadow.

A **[Foreign Set-I, II, III, 2017]**

Sol.



½

In ΔABC, $\frac{AB}{AC} = \tan 60°$

$\frac{h}{x} = \sqrt{3}$

$\Rightarrow$ $h = x\sqrt{3}$ **1**

In ΔABD, $\frac{AB}{AD} = \tan \theta$

$\frac{h}{3x} = \tan \theta$

$\Rightarrow$ $\frac{x\sqrt{3}}{3x} = \frac{1}{\sqrt{3}} = \tan 30°$ **1**

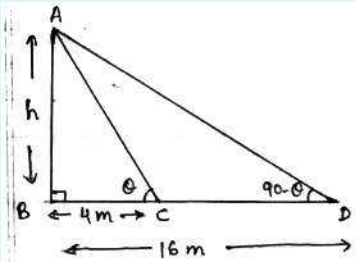
$\therefore$ $\theta = 30°$ ½

[CBSE Marking Scheme, 2017]

Q. 4. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower. A [CBSE OD Set-I, II, III, 2017]

Topper Answer, 2017

Sol.



It is given that ∠ACB and ∠ADB are complementary.

Let them be θ and 90-θ respectively.

Now,

In right Δ ABC,

$\tan\theta = \frac{AB}{BC} = \frac{h}{4}$

$\tan\theta = \frac{h}{4}$ — (1)

In right Δ ABD,

$\tan(90-\theta) = \frac{AB}{BD} = \frac{h}{16}$

$\cot\theta = \frac{h}{16}$

$\tan\theta = \frac{16}{h}$ — (2)

$\tan(90-\theta) = \cot\theta$

$\cot\theta = \frac{1}{\tan\theta}$

From (1) and (2),

$\tan\theta = \frac{h}{4} = \frac{16}{h}$

$h^2 = 4 \times 16$

$h = \sqrt{4 \times 16}$

$\therefore h = 2 \times 4$

$h = 8$ m (ignoring -ve value).

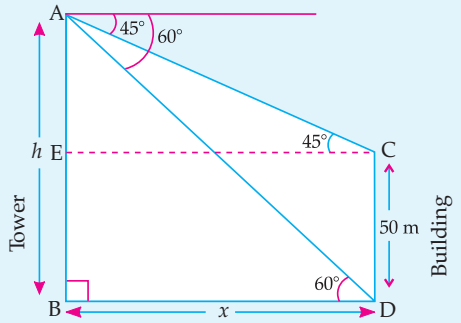
∴ height of tower is 8 m.

3

Q. 5. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3} = 1.73$)

A [CBSE Delhi Set I, II, III, 2016]

Sol.



$\tan 45° = \frac{h-50}{x}$

$\Rightarrow x = h - 50$ ½

$\tan 60° = \frac{h}{x}$ ½

$\Rightarrow x = \frac{h}{\sqrt{3}}$

Hence $h - 50 = \frac{h}{\sqrt{3}}$ ½

$\sqrt{3}h - 50\sqrt{3} = h$

$\sqrt{3}h - h = 50\sqrt{3}$

$h(\sqrt{3}-1) = 50\sqrt{3}$

$h = \frac{50\sqrt{3}}{\sqrt{3}-1}$ ½

$h = \frac{50\sqrt{3}(\sqrt{3}+1)}{3-1}$

$h = \frac{50(3+\sqrt{3})}{2}$

$\Rightarrow h = 75 + 25\sqrt{3} = 75 + 43.25$

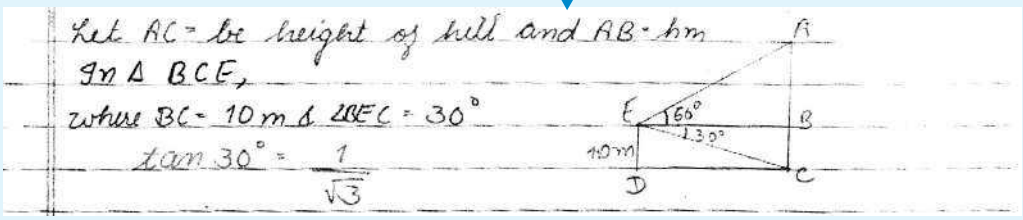
$= 118.25$ m. 1

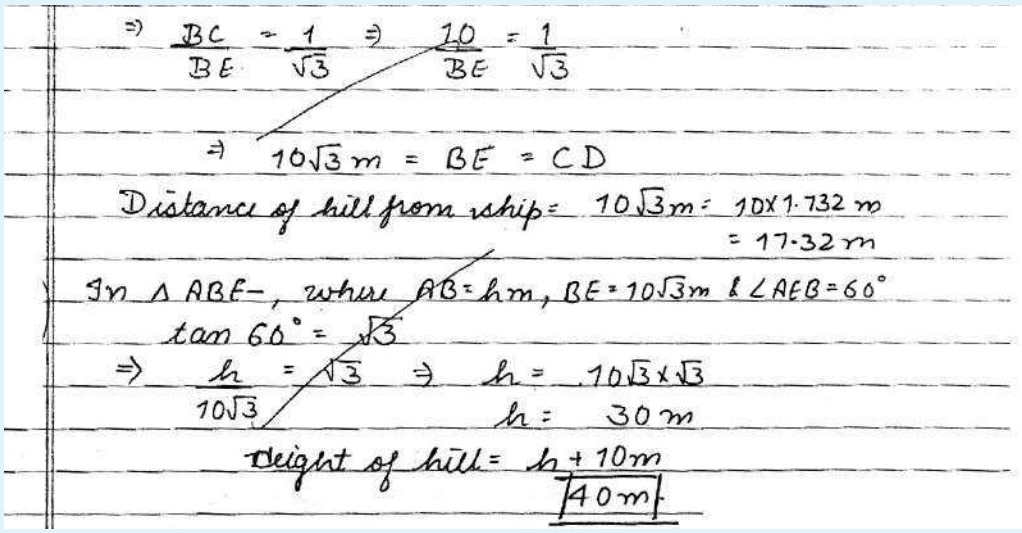
[CBSE Marking Scheme, 2016]

Q. 6. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill. A [CBSE OD, Set-II, 2016]

Topper Answer, 2016

Sol.





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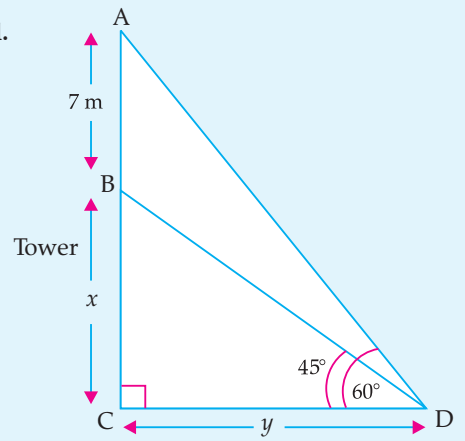
Q. 7. A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the tower correct upto one place of decimal. (Use $\sqrt{3} = 1.73$)

A [Foreign Set II, 2016]

Sol.



½

In ΔBCD, $\frac{BC}{CD} = \tan 45°$

i.e., $\frac{x}{y} = \tan 45° = 1$

$\Rightarrow \quad x = y$ **1**

In ΔACD, $\frac{x+7}{y} = \tan 60° = \sqrt{3}$

$\Rightarrow \quad y\sqrt{3} = x + 7$

Putting $y = x$, then

$x\sqrt{3} = x + 7$

$\Rightarrow \quad 7 = (\sqrt{3} - 1)x$ **1+½**

$x = \frac{7(\sqrt{3}+1)}{2}$

$= \frac{7(2.73)}{2}$

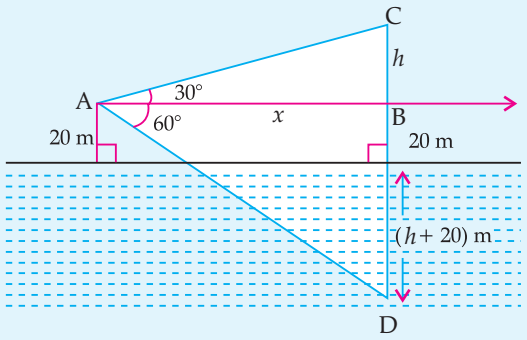
$= \frac{19.21}{2} = 9.60$

$= 9.6$ m

[CBSE Marking Scheme, 2016]

AI **Q. 8. At a point A, 20 metre above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A ?** A [CBSE OD Set-I, II, III, 2015]

Sol.



From, ΔABC,

$$\frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \sqrt{3}h. \qquad \text{...(i) ½}$$

From ΔABD,

$$\frac{40+h}{x} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad x = \frac{40+h}{\sqrt{3}} \qquad \text{...(ii) ½}$$

From (i) and (ii),

$$\therefore \quad \sqrt{3}\,h = \frac{40+h}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = 40 + h \text{ m.}$$

$$\Rightarrow \quad h = 20 \text{ m.} \qquad \mathbf{1}$$

$$\therefore \quad x = 20\sqrt{3} \text{ m}$$

$$\therefore \quad AC = \sqrt{(BC)^2 + (AB)^2}$$

$$= \sqrt{(20)^2 + (20\sqrt{3})^2}$$

$$= \sqrt{400 + 1200}$$

$$= 40 \text{ m.} \qquad \mathbf{1}$$

Hence, the distance of the cloud = 40 m.

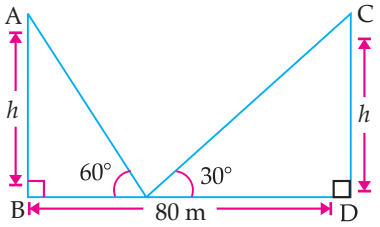
[CBSE Marking Scheme, 2015]

Long Answer Type Questions — 5 marks each

AI **Q. 1. The two palm trees are of equal heights and are standing opposite to each other on either side of the river, which is 80 m wide. From a point O between them on the river the angles of elevation of the top of the trees are 60° and 30°, respectively. Find the height of the trees and the distances of the point O from the trees. (use $\sqrt{3} = 1.73$)**

A **[CBSE SQP, 2020-21]**

Sol.



1

Let BD = width of river = 80 m ½

$AB = CD$

= height of both palm trees = h

$BO = x$ ½

$OD = 80 - x$

In ΔABO,

$$\tan 60° = \frac{h}{x} \qquad \text{½}$$

$$\sqrt{3} = \frac{h}{x} \qquad \text{...(i) ½}$$

$$h = \sqrt{3}x$$

In ΔCDO,

$$\tan 30° = \frac{h}{(80-x)} \qquad \text{½}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{(80-x)} \qquad \text{...(ii) ½}$$

Solving (i) and (ii), we get

$x = 20$

$h = \sqrt{3}x$ [From eqn. (i)]

$= 1.73 \times 20$

$= 34.6$

The height of the trees = h = 34.6 m ½

$BO = x = 20$ m

$DO = 80 - x$

$= 80 - 20$

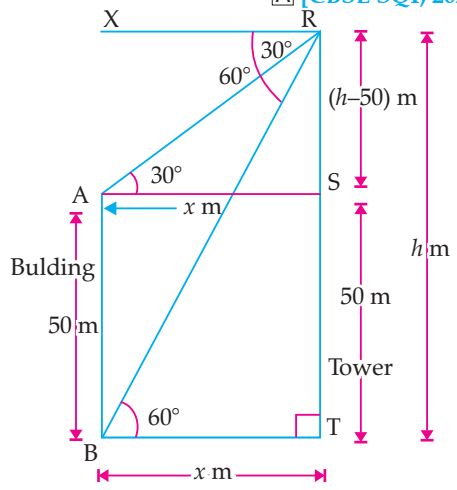
$= 60$ m ½

∴ The distances of the point O from the trees are 20 m and 60 m respectively.

AI **Q. 2. The angles of depression of the top and bottom of a building 50 meters high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower, and also the horizontal distance between the building and the tower.**

A **[CBSE SQP, 2020-21]**

Sol.



1

Let, height of building

$AB = 50$ m

and height of tower,

$RT = h$ m ½

$BT = AS = x$ m

$AB = ST = 50$ m ½

$RS = TR - TS = (h - 50)$ m

In ΔARS, $\tan 30° = \frac{RS}{AS}$

$\frac{1}{\sqrt{3}} = \frac{h-50}{x}$ ...(i) ½

In ΔRBT, $\tan 60° = \frac{RT}{BT}$

$\sqrt{3} = \frac{h}{x}$ ½

Solving (i) and (ii), we get

$h = 75$ ½

From (ii), $x = \frac{h}{\sqrt{3}} = \frac{75}{\sqrt{3}}$ ½

$= 25\sqrt{3}$

Hence, height of the tower = 75 m ½

Distance between the building and the tower

$= 25\sqrt{3}$

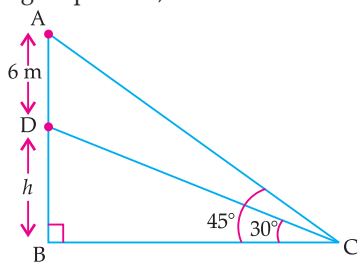
$= 25 \times 1.732$

$= 43.30$ m ½

AI **Q. 3. A vertical tower stands on horizontal plane and is surmounted by a vertical flag-staff of height 6 m. The angles at a point on the bottom and top of the flag-staff with the ground are 30° and 45° respectively. Find the height of the tower.**

(Take $\sqrt{3} = 1.73$) U **[CBSE Delhi Set-I, 2020]**

Sol. According to question,



1

AD is a flagstaff and BD is a tower.

In ΔABC,

$\tan 45° = \frac{AB}{BC}$ ½

$\Rightarrow 1 = \frac{h+6}{BC}$

$\Rightarrow BC = h + 6$ ...(i) ½

In ΔDBC,

$\tan 30° = \frac{DB}{BC}$ ½

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{h+6}$ [from (i)]

$\Rightarrow h\sqrt{3} = h + 6$ ½

$\Rightarrow h\sqrt{3} - h = 6$

$\Rightarrow h(\sqrt{3} - 1) = 6$

$\Rightarrow h = \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ ½

$\Rightarrow h = \frac{6(\sqrt{3}+1)}{2}$

$\Rightarrow h = 3(\sqrt{3}+1)$

$= 3(1.73 + 1)$ ½

$\Rightarrow h = 3 \times 2.73$

$\Rightarrow h = 8.19$ m.

$\therefore$ The height of the tower is 8.19 m. **1**

Q. 4. From a point on the ground, the angles of elevation of the bottom and the top of a tower are 45° and 60° respectively. Find the height of the tower.

A **[CBSE OD, Set-I, 2020]**

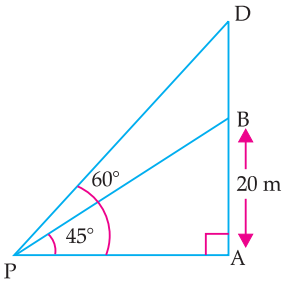
Sol. Let the height of the tower be BD

In ΔPAB,

$\tan 45° = \frac{AB}{AP}$ **1**

$\Rightarrow 1 = \frac{20}{AP}$

$\Rightarrow AP = 20$ m



1

In ΔPAD,

$\tan 60° = \frac{AD}{AP} = \frac{20 + BD}{20}$ **1**

$\Rightarrow \sqrt{3} = \frac{20 + BD}{20}$

$\Rightarrow 20 + BD = 20\sqrt{3}$

$\Rightarrow BD = 20\sqrt{3} - 20$

$= 20(\sqrt{3} - 1)$ **1**

$= 20\,(1.732 - 1)$

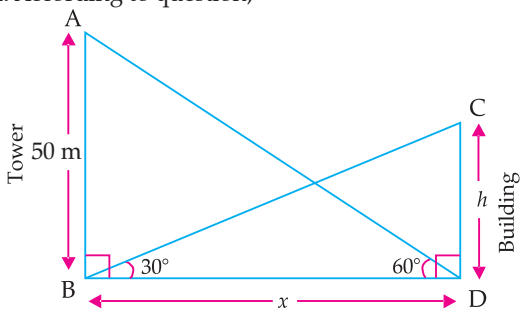
$= 20 \times 0.732$

$= 14.64$ m.

Hence, the height of the tower is 14.64 m. **1**

Q. 5. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of a tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building. A **[CBSE OD, Set-II, 2020]**

Sol. According to question,



1

In ΔABD

$$\tan 60° = \frac{AB}{BD}$$ 1

$$\Rightarrow \quad \sqrt{3} = \frac{50}{BD}$$

$$\Rightarrow \quad BD = \frac{50}{\sqrt{3}}$$ ½

Now in ΔBDC,

$$\tan 30° = \frac{CD}{BD}$$ 1

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}} = \frac{h\sqrt{3}}{50}$$ ½

$$\Rightarrow \quad 3h = 50$$

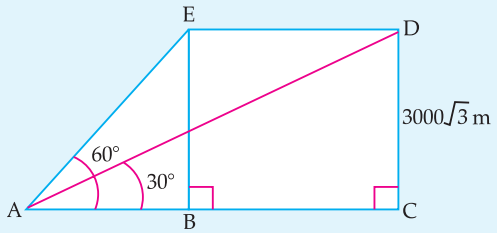
$$\Rightarrow \quad h = \frac{50}{3} = 16.67$$

Hence, the height of the building is 16.67 m. 1

AI **Q. 6. The angle of elevation of an air plane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the air plane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.**

A [CBSE SQP, 2020]

Sol.



Correct figure 1

In ΔABE, $\frac{BE}{AB} = \tan 60°$ 1

$$\Rightarrow \quad AB = 3000 \text{ m}$$

In ΔDAC, $\frac{DC}{AC} = \tan 30°$ 1

$$\Rightarrow \quad AC = 9000 \text{ m}$$

$$BC = AC - AB = 6000 \text{ m}$$ 1

$$\therefore \text{ Speed of aeroplane} = \frac{6000 \text{ m}}{30 \text{ s}} = 200 \text{ m/sec.}$$ 1

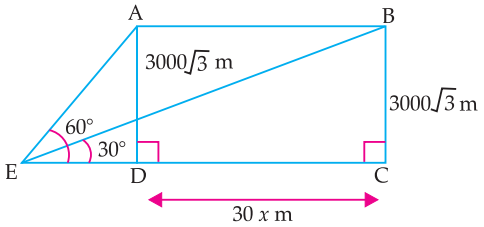
[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

$$\angle AED = 60° \text{ and } \angle BEC = 30°$$

$$AD = BC = 3000\sqrt{3} \text{ m}$$

Let the speed of the aeroplane = x m/s



1

Then, $AB = DC = 30 \times x$

$= 30x$ m ...(i) 1

In ΔAED, $\angle D = 90°$

$$\tan 60° = \frac{AD}{DE}$$

$$\sqrt{3} = \frac{3000\sqrt{3}}{DE}$$

$DE = 3000$ m ...(ii) 1

In ΔBEC, $\angle C = 90°$

$$\tan 30° = \frac{BC}{EC}$$

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{DE + CD}$$

$DE + CD = 3000 \times 3$

$3000 + 30x = 9000$ (from (i) and (ii))

$30x = 6000$

$x = 200$ m/s

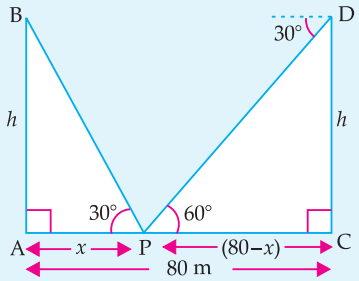
Hence, the speed of plane = 200 m/s. 1

$$= 200 \times \frac{18}{5} = 720 \text{ km/h}$$ 1

AI **Q. 7. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point in between them on the road, the angles of elevation of the top of poles are 60° and 30° respectively. Find the height of the poles and the distances of the point P from the poles.**

A [CBSE Delhi, Set-I, 2019]

Sol.



1

In ΔABP, $\frac{h}{x} = \tan 60° = \sqrt{3}$ ...(i) ½

In ΔCDP, $\frac{h}{80 - x} = \tan 30° = \frac{1}{\sqrt{3}}$ ...(ii) ½

Dividing (i) by (ii), we get

$$\frac{80-x}{x} = \frac{3}{1}$$

$\Rightarrow \quad 3x = 80 - x$

or $\quad 4x = 80$

$\Rightarrow \quad x = 20$ m. **1**

and $\quad h = 20\sqrt{3}$ m. **1**

$\therefore$ Height of poles is $20\sqrt{3}$ m

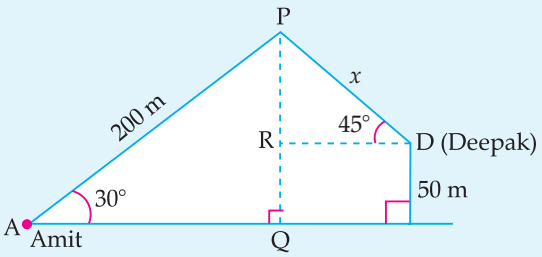
and the distances P are 20 m and 60 m from poles. **1**

[CBSE Marking Scheme, 2019]

AI **Q. 8. Amit, standing on a horizontal plane, and a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, and the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.**

A **[CBSE OD, Set-I, 2019]**

Sol.



1

In ΔAPQ

$$\frac{PQ}{AP} = \sin 30° = \frac{1}{2}$$ **1**

$$PQ = (200)\left(\frac{1}{2}\right) = 100 \text{ m}$$ **1**

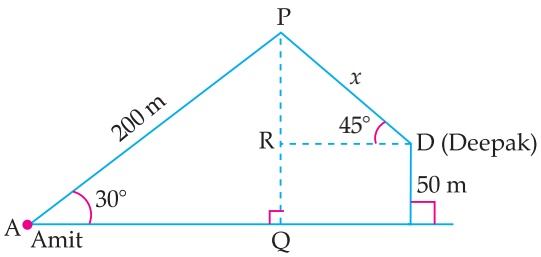
$$PQ = 100 - 50 = 50 \text{ m}$$ **1**

In ΔPRD, $\quad \frac{PR}{PD} = \sin 45° = \frac{1}{\sqrt{2}}$

$$PD = (PR)(\sqrt{2}) = 50\sqrt{2} \text{ m}$$ **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:



1

Let P be the position of the bird, A be the position of Amit, D be the position of Deepak and FD be the building at which Deepak is standing at height 50 m.

Given, $AP = 200$ m and $FD = 50$ m **1**

In ΔPQA, $\quad \angle Q = 90°$

$$\sin 30° = \frac{PQ}{PA}$$ ½

$\Rightarrow \quad \frac{1}{2} = \frac{PQ}{200}$

$\Rightarrow \quad PQ = \frac{200}{2} = 100$ m ½

$\therefore \quad PR = PQ - RQ$

$= PQ - FD$

$= (100 - 50)$ m

$= 50$ m ½

In ΔPRD, $\quad \angle R = 90°$

$$\sin 45° = \frac{PR}{PD}$$ ½

$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{50}{PD}$

$\Rightarrow \quad PD = 50\sqrt{2}$

$= 50 \times 1.414$

$= 70.7$ m

Thus, the distance of the bird from Deepak is 70.7 m. **1**

COMMONLY MADE ERROR

- Many candidates express the answer in 3 significant figures or four significant figures which is not necessary if not asked in questions.

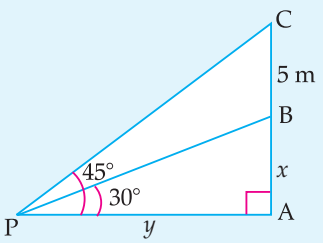
ANSWERING TIP

- Students should practice mathematical calculations each day to reduce calculation errors.

Q. 9. From a point P on the ground, the angle of elevation of the top of a tower is 30° and that of the top of the flag-staff fixed on the top of the tower is 45°. If the length of the flag-staff is 5 m, find the height of the tower. (Use $\sqrt{3} = 1.732$)

A **[CBSE OD, Set-III, 2019]**

Sol. Let AB be a tower and BC be a flagstaff.



1

In ΔPAC, According to question,

$$\frac{AC}{AP} = \tan 45° = 1 \qquad \textbf{1}$$

$$\Rightarrow \quad x + 5 = y \qquad ½$$

In ΔPAB, $\quad \frac{x}{y} = \tan 30° = \frac{1}{\sqrt{3}}$

$$\frac{x}{x+5} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{5}{\sqrt{3}-1}$$

$$= \frac{5(\sqrt{3}+1)}{2}$$

$$= \frac{13.66}{2}$$

$$= 6.83 \qquad \textbf{1½}$$

$\therefore$ Height of tower $= 6.83$ m **1**

[CBSE Marking Scheme, 2019]

COMMONLY MADE ERROR

- Many candidates were not able to understand the language of question.

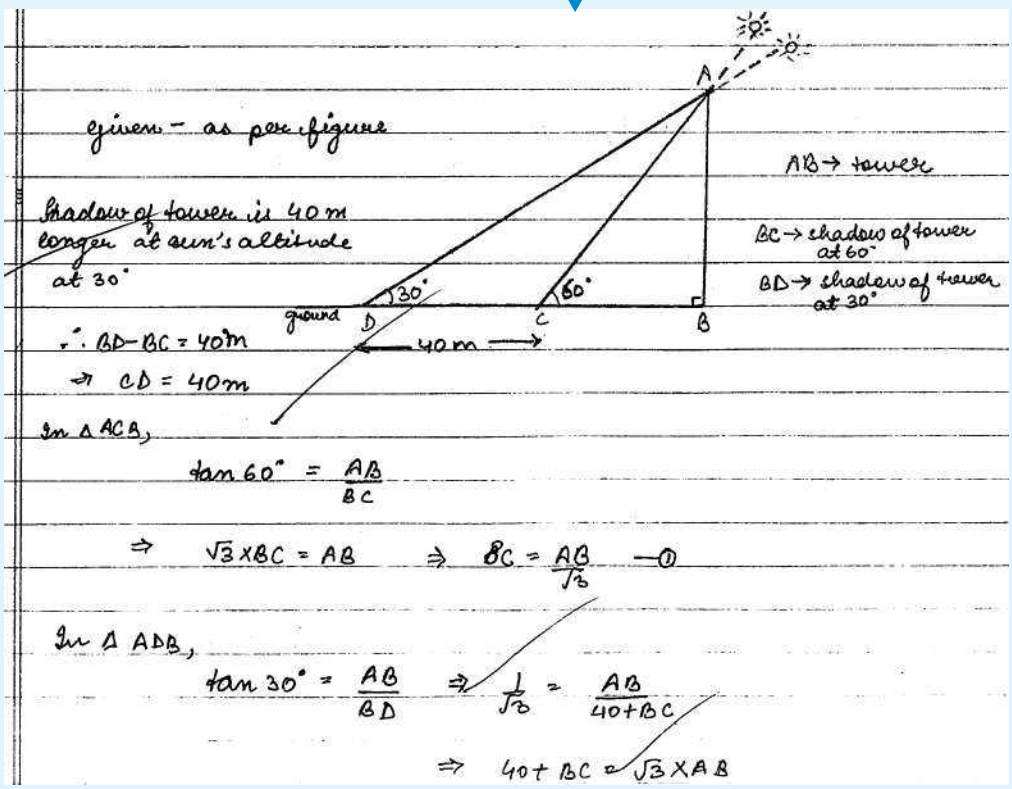
ANSWERING TIP

- Do sufficient practice of drawing correct diagrams for problems based on Heights and Distances.

Q. 10. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower. (Given $\sqrt{3} = 1.732$) [CBSE Delhi Term, 2019]

Topper Answer, 2019

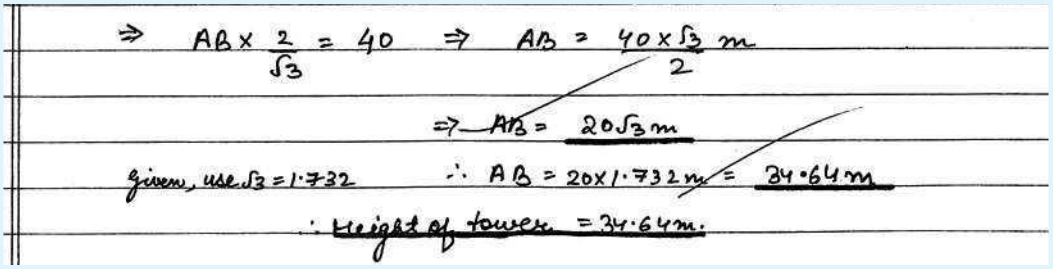
Sol.



$\Rightarrow \quad 40 + \frac{AB}{\sqrt{3}} = \sqrt{3} \times AB$ [Put $BC = \frac{AB}{\sqrt{3}}$ from ①].

$\Rightarrow \quad AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 40$

$\Rightarrow \quad AB\left(\frac{3-1}{\sqrt{3}}\right) = 40$

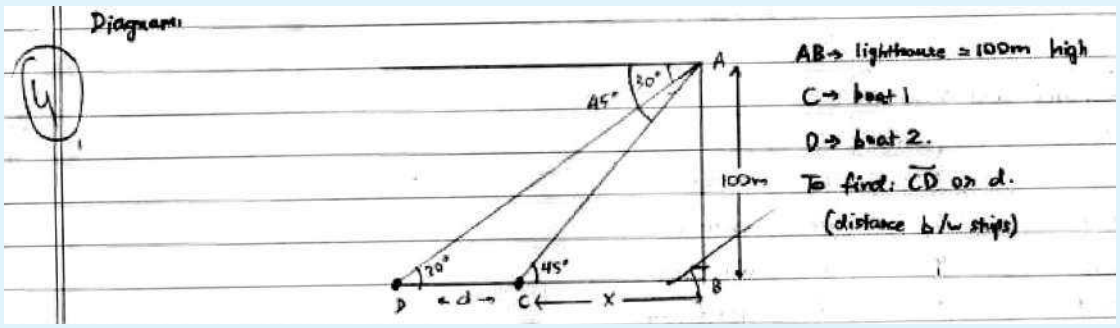


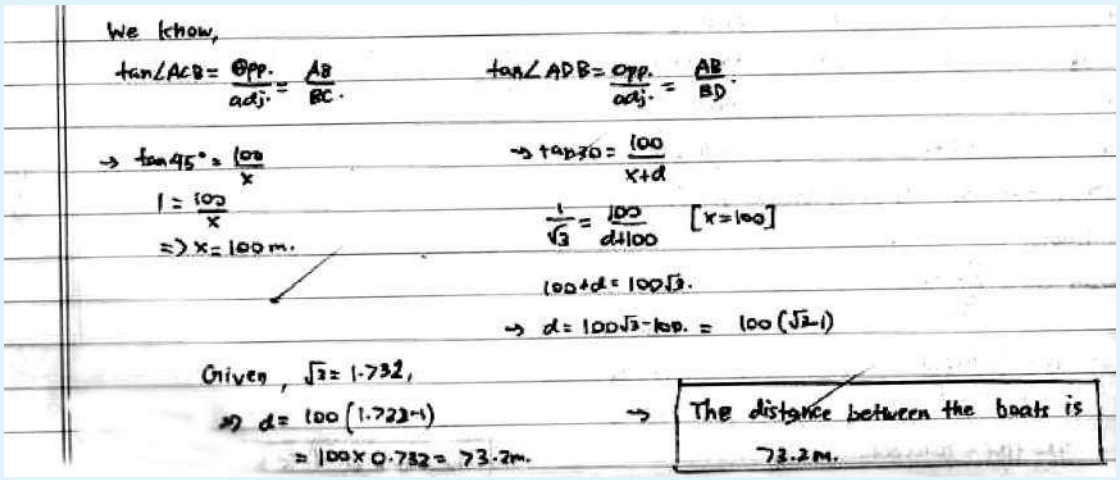
5

AI **Q. 11. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3}$ = 1·732]** C + A **[CBSE Delhi/OD, 2018]**

Topper Answer, 2018

Sol.





5

COMMONLY MADE ERROR

- Most candidates are unable to draw the diagram as per the given data and lose their marks. Some candidates do calculation errors while putting the values of $\sqrt{3}$ = 1.73 instead of 1.732 and hence write inaccurate answer.

ANSWERING TIP

- Students should do rounding off at the end while calculating the final answer.

Q. 12. A man on the top of a vertical observation tower observes a car moving at uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how long will the car take to reach the observation tower from this point ? C + A **[CBSE SQP, 2018]**

Sol. Let the speed of car by x m/minute

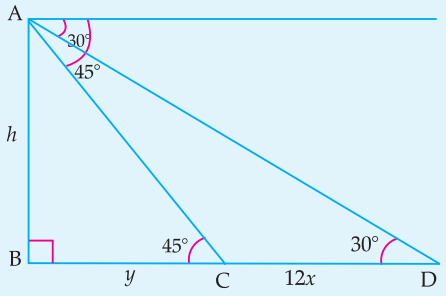
In ΔABC,

$$\frac{h}{y} = \tan 45° \qquad \textbf{1}$$

$$\Rightarrow \quad h = y \qquad \frac{1}{2}$$

In ΔABD, $\quad \frac{h}{y+12x} = \tan 30° \qquad \textbf{1}$

$$\Rightarrow \quad h\sqrt{3} = y + 12x \qquad \frac{1}{2}$$



1

$$y\sqrt{3} - y = 12x$$

$$y = \frac{12x}{\sqrt{3}-1} = \frac{12x(\sqrt{3}+1)}{2}$$

$$\Rightarrow \quad y = 6x(\sqrt{3}+1)$$

Hence, time taken from C to $B = 6(\sqrt{3}+1)$ minutes

[CBSE Marking Scheme, 2018] 1

Detailed Solution:

Let AB be the tower of height h and x be the distance between two cars

$$\angle AQB = 45°$$

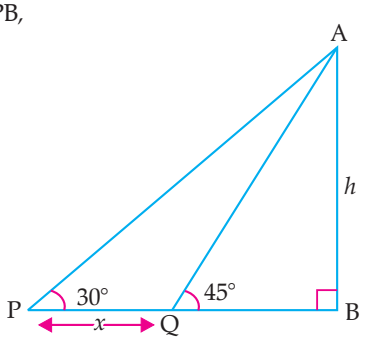
Now, in ΔABQ,

$$\tan 45° = \frac{AB}{BQ} \qquad \textbf{1}$$

$$\Rightarrow \quad 1 = \frac{h}{BQ}$$

$$\Rightarrow \quad BQ = h$$

In ΔAPB,



1

$$\tan 30° = \frac{AB}{PB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+h}$$

$$\Rightarrow \quad x + h = h\sqrt{3}$$

$$i.e., \quad x = h(\sqrt{3}-1)$$

Thus, $\quad \text{Speed} = \frac{h(\sqrt{3}-1)}{12}$ m/min **1**

Time for remaining distance, **1**

$$= \frac{h}{\frac{h(\sqrt{3}-1)}{12}}$$

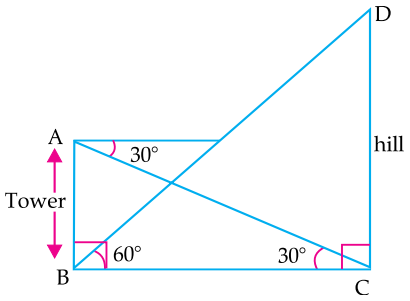
$$= \frac{12(\sqrt{3}+1)}{3-1}$$

$$= \frac{12}{2}(\sqrt{3}+1)$$

$$= 6(\sqrt{3}+1) \text{ min} \qquad \textbf{1}$$

Q. 13. The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of depression from the top of the tower of the foot of the hill is 30°. If tower is 50 meter high, find the height of the hill. A **[CBSE Comptt. Set-I, II, III, 2018]**
[CBSE Delhi Set-I, II, III, 2015]

Sol.



Let $AB = 50$ m be the height of the tower and CD be the height of hill. **1**

Now, in ΔABC,

$$\angle ABC = 90°$$

$$\tan 30° = \frac{AB}{BC} \qquad \textbf{1}$$

or, $\quad BC = \frac{50}{\tan 30°} = \frac{50\times\sqrt{3}}{1}$ m

or, $\quad BC = 50\sqrt{3}$ m **1**

Again in ΔBCD, $\angle BCD = 90°$

$$\tan 60° = \frac{DC}{BC}$$

or, $\quad DC = BC\tan 60°$ **1**

$$= 50\sqrt{3}\times\sqrt{3} \text{ m}$$

$$\Rightarrow \quad DC = 150 \text{ m}$$

$\therefore$ The height of hill is 150 m. **1**

COMMONLY MADE ERROR

- The concept of angle of depression is not clear to many students. That's why they are not able to draw the diagram correctly.

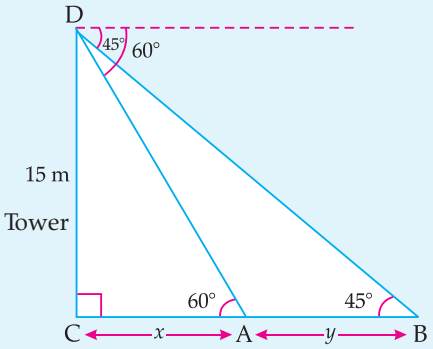
ANSWERING TIP

- The concept of angle of depression and angle of elevation must be clear to the students.

Q. 14. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

C + A [CBSE Delhi Set-I, 2017]

Sol.



1

In ΔDCA, $\frac{DC}{CA} = \tan 60°$

$\Rightarrow \frac{15}{x} = \sqrt{3}$ ½

$\Rightarrow x = \frac{15}{\sqrt{3}}$

$\Rightarrow x = 5\sqrt{3}$ **1**

In ΔDCB, $\frac{DC}{CB} = \tan 45° = \frac{15}{x+y} = 1$ ½

$\Rightarrow x + y = 15$ **1**

$\Rightarrow 5\sqrt{3} + y = 15$

$\Rightarrow y = 15 - 5\sqrt{3}$

$= 5(3 - \sqrt{3})$ m

Hence, the distance between the points

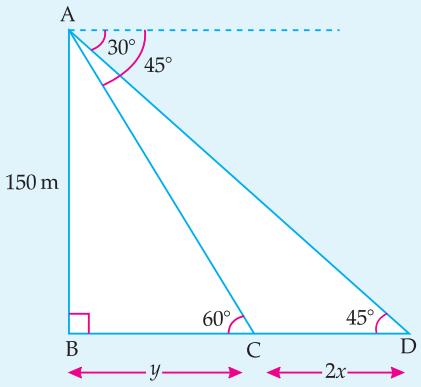
$= 5(3 - \sqrt{3})$ m.

[CBSE Marking Scheme, 2017] **1**

Q. 15. A moving boat observed from the top of a 150 m high cliff, moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

A [CBSE Delhi Set-I, 2017]

Sol.



1

Let the speed of the boat be x m/min.

$\therefore$ Distance covered in 2 minutes $= 2x$

$\therefore \quad CD = 2x$

Let BC be y m.

In ΔABC, $\frac{AB}{BC} = \tan 60°$ ½

$\Rightarrow \frac{150}{y} = \sqrt{3}$

$\Rightarrow y = \frac{150}{\sqrt{3}}$

$\Rightarrow y = 50\sqrt{3}$ m. ...(i)

In ΔABD, $\frac{AB}{BD} = \tan 45°$ ½

$\Rightarrow \frac{150}{y+2x} = 1$

$\Rightarrow y + 2x = 150$ ...(ii) **1**

Substituting the value of y from (i) in (ii),

$50\sqrt{3} + 2x = 150$

$2x = 150 - 50\sqrt{3}$

$2x = 50(3 - \sqrt{3})$

$x = 25(3 - \sqrt{3})$ m. **1**

Speed of the boat $= 25(3 - \sqrt{3})$ m/min.

$= \frac{25(3 - \sqrt{3}) \times 60}{1000}$

$= \frac{3}{2}(3 - \sqrt{3})$ km/h

$= 19.02$ km/h **1**

[CBSE Marking Scheme, 2017]

Q. 16. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. If both the ships are in the same line that one ship is exactly behind the other, find the distance between the ships. C + A [Foreign Set-II 2017]

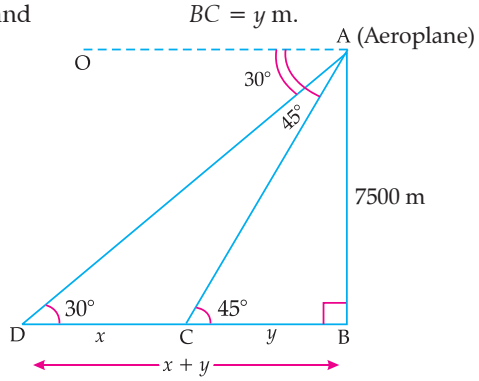
Sol. Let AB be the height of the aeroplane, then AB = 7500 m.

Also let D and C be the positions of two ships on the same line. From the point A of an aeroplane, the angles of depression of two ships D and C are $\angle OAD = 30°$ *i.e.*, $\angle BDA = 30°$ and $\angle OAC = 45°$ *i.e.*, $\angle BCA = 45°$

Let distance between two ships

$DC = x$ m

and $BC = y$ m.



1

In ΔABC, $\frac{AB}{BC} = \tan 45°$ ½

$\Rightarrow \frac{7500}{y} = 1$

$\Rightarrow y = 7500$ **1**

In ΔABD, $\frac{AB}{BD} = \tan 30°$ ½

$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$

$\Rightarrow x + y = 7500\sqrt{3}$

$x + 7500 = 7500\sqrt{3}$ ½

$x = 7500\sqrt{3} - 7500$

$= 7500\left(\sqrt{3} - 1\right)$

$= 7500(1.732 - 1)$

$= 7500 \times 0.732$

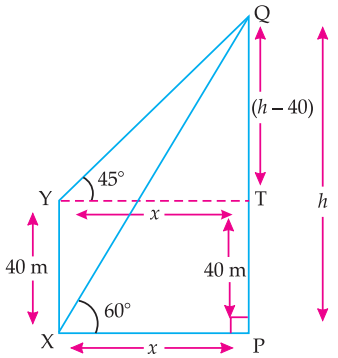
$= 5490$ m

Hence, the distance between two ships

$= 5490$ m **1½**

Q. 17. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3} = 1.73$) U **[CBSE OD Set I, III, 2016]**

Sol.



1

Let $PX = x$ m and $PQ = h$ m

$\therefore QT = (h - 40)$ m

In ΔPQX,

$\tan 60° = \frac{h}{x}$ ½

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$ ...(i) **1**

In ΔQTY, $\tan 45° = \frac{h-40}{x}$ ½

$\Rightarrow 1 = \frac{h-40}{x}$

$\Rightarrow x = h - 40$ ...(ii) **1**

Solving (i) and (ii),

$x = \sqrt{3}x - 40$

$\left(\sqrt{3}x - x\right) = 40$

or $\left(\sqrt{3} - 1\right)x = 40$

or $x = \frac{40}{\sqrt{3}-1} = 20\left(\sqrt{3}+1\right)$ m

$\therefore h = \sqrt{3} \times 20\left(\sqrt{3}+1\right) = 20\left(3+\sqrt{3}\right)$ m **1**

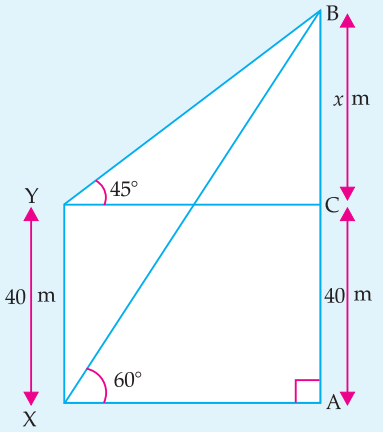
$20(3 + 1.73) = 20 \times 4.73$

Hence, the height of tower is 94.6 m.

Q. 18. The angle of elevation of the top B of a tower AB from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Find the height of the tower AB and the distance XB. A **[CBSE SA-2, 2016]**

Sol. In ΔYCB, we have

$\tan 45° = \frac{BC}{YC}$



1

$1 = \frac{x}{YC}$

$YC = x$ m

$\Rightarrow XA = x$ m

In ΔXAB,

$\tan 60° = \frac{AB}{XA}$ ½

$$\sqrt{3} = \frac{x+40}{x} \quad ½$$

$$\sqrt{3}x = x + 40$$

$$x\sqrt{3} - x = 40$$

$$x = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \quad ½$$

$$= 20(\sqrt{3}+1)$$

$$= \left(20\sqrt{3}+20\right) \text{ m}$$

$\therefore$ Height of the tower, $AB = x + 40$

$$= 20\sqrt{3} + 20 + 40 \quad ½$$

$$= 20\sqrt{3} + 60$$

$$= 20(\sqrt{3} + 3) \text{ m} \quad ½$$

$$20 \times 4.732 = 94.64 \text{ m}$$

In ΔXAB,

$$\sin 60° = \frac{AB}{BX} \quad ½$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{BX} \quad ½$$

$$BX = \frac{20(\sqrt{3}+3)2}{\sqrt{3}}$$

$$= 40\left(\sqrt{3}+1\right) \text{ m}$$

$$= 40 \times 2.732 \text{ m}$$

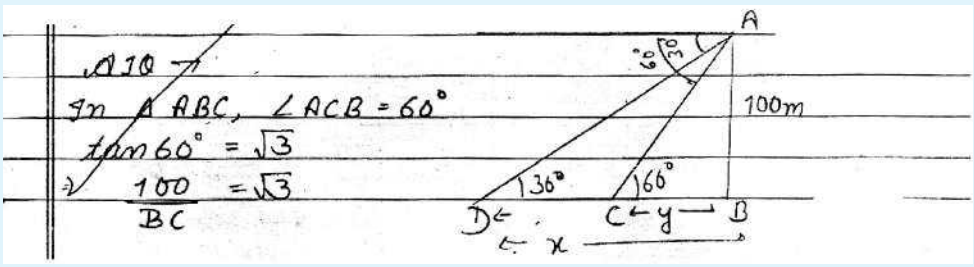
$$= 109.28 \text{ m} \quad ½$$

[CBSE Marking Scheme, 2016]

Q. 19. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1.73$) C [CBSE OD, Set-II, 2016]

Topper Answer, 2017

Sol.



$\frac{100}{\sqrt{3}} = BC = y$

In ΔABD, $\angle ADB = 30°$

$\tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{AB}{BD} = \frac{1}{\sqrt{3}} \Rightarrow \frac{100}{BD} = \frac{1}{\sqrt{3}}$

$100\sqrt{3} = BD = x$

Required distance travelled by ship $= (y - x) = x - y$

$= 100\sqrt{3} - \frac{100}{\sqrt{3}}$ m

$x - y \Rightarrow 100\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = 100\left[\frac{3-1}{\sqrt{3}}\right] = \frac{100 \times 2}{\sqrt{3}}$

$CD = x - y \Rightarrow \frac{100 \times 2 \times \sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{200\sqrt{3}}{3}$ m

$CD = \frac{200 \times 1.73}{3}$ m $= \frac{346}{3}$ m

$\Rightarrow$ 115.33 m

5

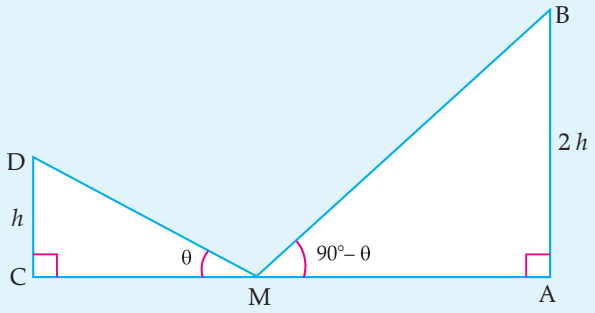
Q. 20. Two posts are k metre apart and the height of one is double that of the other. If from the mid-point of the line segment joining their feet, an observer finds the angles of elevation of their tops to be complementary, then find the height of the shorter post. A [CBSE Term-2, 2015]

Sol. Let AB and CD be the two posts such that

$AB = 2\,CD$.

Let M be the mid-point of CA.

Let $\angle CMD = \theta$ and $\angle AMB = 90° - \theta$



1

Clearly, $CM = MA = \frac{1}{2}k$

Let $CD = h$ m, then AB = 2h m

Now, $\frac{AB}{AM} = \tan(90° - \theta) = \cot\theta$ **1**

$\Rightarrow \quad \frac{2h}{\left(\frac{k}{2}\right)} = \cot\theta$

$\Rightarrow \quad \frac{2h}{\left(\frac{k}{2}\right)} = \cot\theta$

$\Rightarrow \quad \cot\theta = \frac{4h}{k}$ ...(i) **1**

Also in ΔCMD, $\frac{CD}{CM} = \tan\theta$

$\Rightarrow \quad \frac{h}{\frac{k}{2}} = \tan\theta$

$\Rightarrow \quad \tan\theta = \frac{2h}{k}$ ...(ii) **1**

Multiplying (i) and (ii), $\frac{4h}{k} \times \frac{2h}{k} = 1$

$\therefore \quad h^2 = \frac{k^2}{8}$

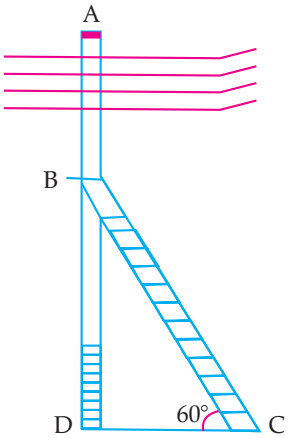
$\Rightarrow \quad h = \frac{k}{2\sqrt{2}} = \frac{k\sqrt{2}}{4}$ m **1**

[CBSE Marking Scheme, 2015]

Visual Case Based Questions — 4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

AI **Q. 1. An electrician has to repaired an electric fault on the pole of height 5 m. She needs to reach a point 1.3 m below the top of the pole to undertake the repair work (see figure)**



(i) What is the length of BD ?

(a) 1.3 m **(b) 5 m**

(c) 3.7 m **(d) None of these**

Sol. Correct option: **(c)**.

Explanation: From figure, the electrician is required to reach at the point B on the pole AD.

So, $BD = AD - AB$

$= (5 - 1.3)$ m $= 3.7$ m

(ii) What should be the length of Ladder, when inclined at an angle of 60° to the horizontal ?

(a) 4.28 m **(b)** $\frac{3.7}{\sqrt{3}}$ **m**

(c) 3.7 m **(d) 7.4 m**

Sol. Correct option: **(a)**.

Explanation: In ΔADC,

$\sin 60° = \frac{BD}{BC}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3.7}{BC}$

$\Rightarrow \quad BC = \frac{3.7 \times 2}{\sqrt{3}}$

$\Rightarrow \quad BC = 4.28$ m (approx)

(iii) How far from the foot of pole should she place the foot of the ladder ?

(a) 3.7 **(b) 2.14**

(c) $\frac{1}{\sqrt{3}}$ **(d) None of these**

Sol. Correct option: **(b)**.

Explanation: In ΔBDC,

$\therefore \quad \cot 60° = \frac{DC}{BD}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{DC}{3.7}$$

$$\Rightarrow \quad DC = \frac{3.7}{\sqrt{3}}$$

$$\Rightarrow \quad DC = 2.14 \text{ m (approx)}$$

(iv) If the horizontal angle is changed to 30°, then what should be the length of the ladder ?

(a) 7.4 m **(b) 3.7 m**

(c) 1.3 m **(d) 5 m**

Sol. Correct option: **(a).**

Explanation: *In* ΔBDC,

$$\therefore \quad \sin 60° = \frac{BD}{BC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{3.7}{BC}$$

$$\Rightarrow \quad BC = 3.7 \times 2 = 7.4 \text{ m}$$

(v) What is the value of ∠B ?

(a) 60° **(b) 90°**

(c) 30° **(d) 180°**

Sol. Correct option: **(c).**

Explanation: In ΔADC, angle D is 90°.

So, by angle sum property.

$$\angle B + \angle D + \angle C = 180°$$

or, $$\angle B = 180° - (90° + 60°)$$

$$= 30°$$

Q. 2. A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kings way), is about 138 feet (42 metres) in height.

(i) What is the angle of elevation if they are standing at a distance of 42 m away from the monument ?

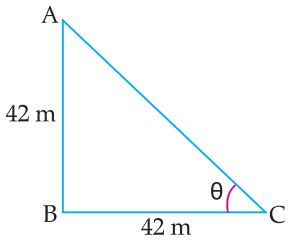
(a) 30° **(b) 45°**

(c) 60° **(d) 0°**

Sol. Correct option: **(b).**

Explanation: Height of Indian gate = 42 m

Distance between students and Indian gate = 42 cm



Now, $$\tan \theta = \frac{AB}{BC}$$

$$\tan \theta = \frac{42}{42}$$

$$\tan \theta = 1$$

$$\tan \theta = \tan 45°$$

$$\theta = 45°$$

Hence, angle of elevation = 45°

(ii) They want to see the tower at an angle of 60°. So, they want to know the distance where they should stand and hence find the distance.

(a) 25.24 m **(b) 20.12 m**

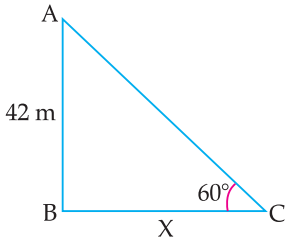
(c) 42 m **(d) 24.64 m**

Sol. Correct option: **(a).**

Explanation: Height of India gate = 42 cm

Angle = 60°

Let the distance between students and India gate = x m.



Now, $$\tan \theta = \frac{AB}{BC}$$

$$\tan 60° = \frac{42}{x}$$

$$\sqrt{3} = \frac{42}{x}$$

$$x = \frac{42}{\sqrt{3}}$$

$$x = \frac{42 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{42\sqrt{3}}{3}$$

$$= 14\sqrt{3} \text{ m} = 25.24 \text{ m}$$

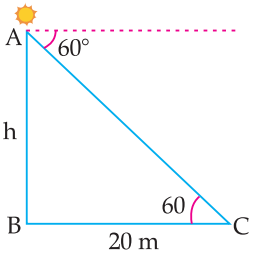
(iii) If the altitude of the Sun is at 60° , then the height of the vertical tower that will cast a shadow of length 20 m is

(a) $20\sqrt{3}$ m **(b) $\frac{20}{\sqrt{3}}$ m**

(c) $\frac{15}{\sqrt{3}}$ m **(d) $15\sqrt{3}$ m**

Sol. Correct option: **(a).**

Explanation:



Let, the height of tower $= h$

Now, $\tan\theta = \frac{AB}{BC}$

$\tan 60° = \frac{h}{20}$

$\sqrt{3} = \frac{h}{20}$

$h = 20\sqrt{3}$

(iv) The ratio of the length of a rod and its shadow is 1 : 1. The angle of elevation of the Sun is

(a) 30° **(b) 45°**

(c) 60° **(d) 90°**

Sol. Correct option: **(b).**

(v) The angle formed by the line of sight with the horizontal when the object viewed is below the horizontal level is

(a) corresponding angle

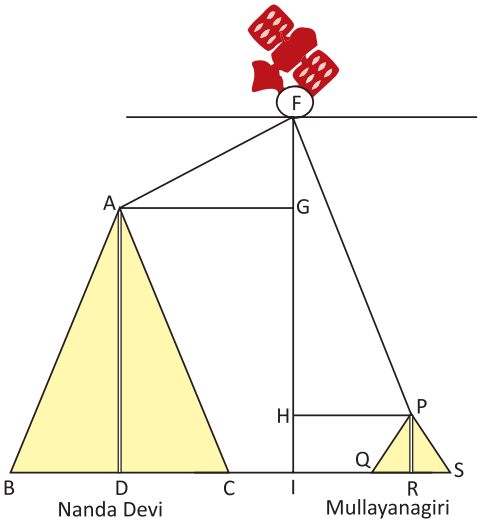
(b) angle of elevation

(c) angle of depression

(d) complete angle

Sol. Correct option: **(a).**

Q. 3. A Satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka ,them being Nanda Devi(height 7,816 m) and Mullayanagiri (height 1,930 m). The angles of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are 30° and 60° respectively. If the distance between the peaks of two mountains is 1937 km, and the satellite is vertically above the midpoint of the distance between the two mountains.



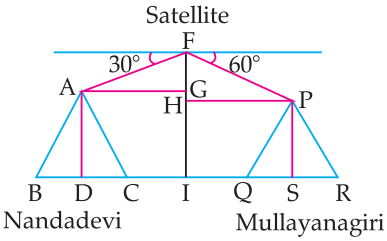
(i) The distance of the satellite from the top of Nanda Devi is

(a) 1118.36 km **(b) 577.52 km**

(c) 1937 km **(d) 1025.36 km**

Sol. Correct option: **(a).**

Explanation:



Now, $AG = \frac{1937}{2}$ km

$\cos\theta = \frac{AG}{AF}$

$\cos 30° = \frac{\frac{1937}{2}}{AF}$

$\frac{\sqrt{3}}{2} = \frac{1937}{2AF}$

$AF = \frac{1937}{\sqrt{3}}$

$AF = 1118.36$ km

(ii) The distance of the satellite from the top of Mullayanagiri is

(a) 1139.4 km **(b) 577.52 km**

(c) 1937 km **(d) 1025.36 km**

Sol. Correct option: **(c).**

Explanation: For ΔFPH,

$\cos\theta = \frac{PH}{FP}$

$\cos 60° = \frac{1937}{2FP}$

$\frac{1}{2} = \frac{1937}{2FP}$

$FP = 1937$ km

(iii) The distance of the satellite from the ground is

(a) 1139.4 km **(b) 577.52 km**

(c) 1937 km **(d) 1025.36 km**

Sol. Correct option: **(b).**

(iv) What is the angle of elevation if a man is standing at a distance of 7816 m from Nanda Devi ?

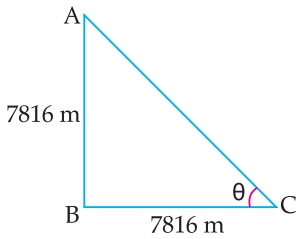
(a) 30° **(b) 45°**

(c) 60° **(d) 0°**

Sol. Correct option: **(b).**

Explanation: Height of Nanda Devi Mountain = 7816 m

Distance between man and mountain = 7816 m.



$$\tan\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{7816}{7816}$$

$$\tan\theta = 1$$

$$\tan\theta = \tan 45°$$

$$\theta = 45°$$

(v) If a mile stone very far away from, makes 45° to the top of Mullanyangiri mountain. So, find the distance of this mile stone form the mountain.

(a) 1118.327 km **(b) 566.976 km**

(c) 1937 km **(d) 1025.36 km**

Sol. Correct option: **(c).**

SELF ASSESSMENT TEST - 5

Maximum Time: 1 hour **MM: 25**

Q. 1. If ΔABC is right angled at C, then find the value of $\cos(A + B)$. C

Q. 2. If $\cos 9\alpha = \sin \alpha$ and $9\alpha < 90°$, then find the value of $\tan 5\alpha$. U

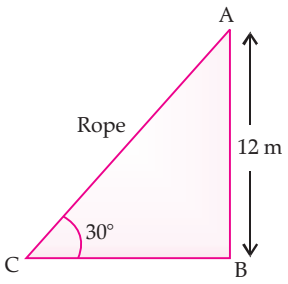
Q. 3. Evaluate: $(\sec A + \tan A)(1 - \sin A)$. A

Q. 4. Find the value of $9 \sec^2 A - 9 \tan^2 A$. U

Q. 5. If the altitude of the Sun is 60°, what is the height of a tower which casts a shadow of length 30 m ?

C + U [CBSE Term-2, Set (B1), 2011]

Q. 6. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 12 m and the angle made by the rope with ground level is 30°.



Give answer of the following questions: C + AE

(i) The distance covered by the artist in climbing the top of the pole is :

(a) 24 m (b) 36 m

(c) 28 m (d) 22 m

(ii) The length of BC is :

(a) $24\sqrt{3}$ m (b) $12\sqrt{3}$ m

(c) $2\sqrt{3}$ m (d) $\sqrt{3}$ m

(iii) If $\sin(A + B) = \frac{\sqrt{3}}{2}$, then the value of (A + B) is:

(a) 30° (b) 90°

(c) 60° (d) 45°

(iv) In ΔABC, given that $\angle A = 60°$ and $\angle C = 30°$, then the value of sin A cos C + cos C sin A is:

(a) 0 (b) ∞

(c) 10 (d) $\frac{3}{2}$

(v) Which mathematical concept is used in this problem ?

(a) Trigonometry (b) Triangle

(c) Circle (d) Mensuration

AI **Q. 7.** If $4 \cos \theta = 11 \sin \theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$. U [CBSE Term-1, 2012] 2

Q. 8. Prove that : $-1 + \frac{\sin A \sin(90° - A)}{\cot(90° - A)} = -\sin^2 A$. 2

U [CBSE Term-1, 2012]

AI **Q. 9.** If $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ and $x \sin \theta = y \cos \theta$, prove that $x^2 + y^2 = 1$.

A [CBSE Term-1, 2011] 3

AI **Q. 10.** Simplify :

$$\frac{\sin\theta \sec(90° - \theta)\tan\theta}{\operatorname{cosec}(90° - \theta)\cos\theta\cot(90° - \theta)} - \frac{\tan(90° - \theta)}{\cot\theta}.$$

U [CBSE Term-1, 2011] 3

Q. 11. Two ships are approaching a light house from opposite directions. The angles of depression of two ships from top of the light house are 30° and 45°. If the distance between two ships is 100 m., find the height of light-house.

A [Foreign set I, II, III, 2014] 5